Critical Analysis of Crack Growth Data

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During the past years, a considerable amount of work has been done in studying the crack growth behavior in composite solid propellants. Experimental findings indicate that a power law relationship exists between the crack growth rate and the mode 1 stress intensity factor. The usefulness of this relationship depends on how accurately this relationship describes the crack growth behavior in the material. Since the crack growth rate and the stress intensity factor are calculated from measured quantities, they are subjected to measurement error and data scatter. Therefore, it is the purpose of this study to review the factors that affect the accuracy and the scatter of the crack growth data and to examine the characterization of the statistical feature af the crack growth behavior.

Introduction

THERE are imperfections existing in solid propellants. These imperfections may be produced during the manufacturing and/or fabricating process of the solid rocket motor. In analyzing the strength of a solid propellant, the localized imperfection may be idealized as a crack in the material. Besides the idealized crack, large cracks can also develop in solid motors during storage, handling, firing, etc. Therefore, to determine the ultimate strength of solid rocket motors, studies should be conducted to determine the significance of flaw type, size, and rate of growth. In order to conduct these studies, a detailed knowledge of crack growth behavior in solid rocket motors is indispensable.

It is well known, on the microscopic scale, that a highly filled composite solid propellant can be considered a nonhomogeneous material. The nonhomogeneous nature of solid propellants is mainly due to the nonuniform distribution of the filler particles, the different sizes of filler particles used, the variation of bond strength between the particles and the binder, and the different crosslink densities of polymer chains. The presence of the filler particles affects not only the bulk properties of the material but also the crack growth behavior. For instance, depending upon the bond strength between the particle and the binder, the fracture path may preferentially either follow or avoid the filler particles, resulting in different fracture surfaces and crack growth rate. In addition to the nonhomogenity of the material, the material may also contain randomly spaced microvoids, incipient damage sites, and microcracks with statistically distributed sizes and directions. Therefore, when this material is strained, highly nonhomogeneous local stress and strength fields can be produced. Hence, the failure location and the degree of damage induced in the material will vary in a random fashion. Because the crack growth behavior is closely related to the local stress and strength fields as well as to the damage state in the material, the variation of these parameters in the material will undoubtedly be a contributing factor to the variability or scatter of crack growth data within a specimen as well as among specimens. In addition to the material-related scatter of crack growth data, the data acquisition and processing methods can also cause scatter of the crack growth data. Hence, the influence of these parameters on the accuracy and the variability of the crack growth rate should be investigated.

During the past years, a considerable amount of work has been done in studying crack growth behavior in solid propellants based on linear viscoelastic fracture mechanics.[1-6] The approach used to study crack growth behavior is to calculate the crack growth rate $\dot{a}$ and the stress intensity factor K_I then to determine the functional relationship between $\dot{a}$ and K_I usefulness of this relationship depends on how accurately this relationship describes the crack growth behavior in the material. Since the crack growth rate and the stress intensity factor are calculated from recorded experimental data, and since solid propellants are highly nonhomogeneous materials (on the microscopic scale), it is expected that the calculated crack growth rate and stress intensity factor will be affected by the error and scatter in the crack length and load measurements. Therefore, it is the purpose of this study to review the factors that affect the accuracy and the scatter of the crack growth data and to examine the characterization of the statistical feature of the crack growth behavior.

Data Collection and Reduction

An experimental test program for studying crack growth behavior in a highly filled composite solid propellant had been conducted by Liu.[5] In the program, centrally cracked sheet specimens of $20.32 \times 5.08 \times 0.508$ cm were used in the crack propagation tests. A 3.81-cm crack was cut through the thickness of the specimen with a razor blade. The geometry of the specimen is shown in Fig. 1.

Crack propagation tests were conducted under a constant crosshead speed af 0.254 cm/min at room temperature. The raw data obtained from the constant rate test were the half crack length a which was measured from the center of the specimen, the time t, and the load p corresponding to the measured crack length. Two half-crack lengths, the left-side crack length, and the right-side crack length were measured at the two ends of the crack, and they are denoted as a_l and a_r. These data served as input to a computer program to calculate the crack growth rates dal/dt, $\dot{a}_l$, and dar/dt, $\dot{a}_r$, as well as the mode 1 stress intensity factor K_I.

There are many factors that can contribute to the variability of test data. Among these factors are specimen preparation,

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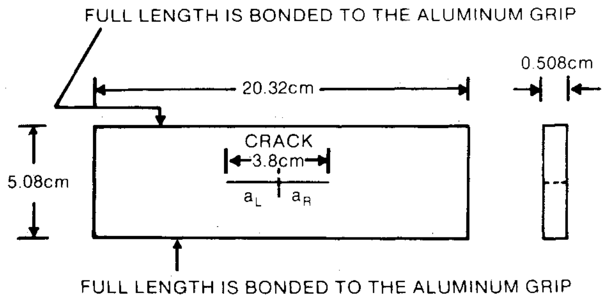

Fig. 1 Specimen geometry.

experimental technique, data acquisition and processing methods, and material property. In this paper, only the effects of data acquisition method, data processing method, and material property on the accuracy and the variability of the crack growth rate are reviewed and discussed.

The determination of crack growth rate $\dot{a}$ requires analysis of the discrete data relating the instantaneous time t to the corresponding crack length a. The general problem encountered in determining the crack growth rate is that of determining the derivative of a function, $a=f(t)$, which is known only at certain data points. Since, on the microscopic scale, a highly filled composite solid propellant can be considered a nonhomogeneous material, it is expected that in addition to the discrete nature of the test data, the test data will show a considerably large scatter. Under this condition, it is anticipated that a smooth and steadily increasing relationship between the crack growth rate and the time is difficult to obtain and that the methods of data collection and processing will affect the accuracy and the scatter of the calculated crack growth rate. In collecting the experimental data, one of the important factors that should be considered is how often the data should be measured. The effect of the time interval that was used to measure the crack length and the methods of $\dot{a}$ calculation on the accuracy of the calculated $\dot{a}$ was investigated by Liu.[6] In Liu's study, the raw data obtained from the constant displacement rate test were the half-crack length a, the time t and the load P corresponding to the measured time. The recorded crack length and time were used to calculate the crack growth rate by four methods. The methods considered were 1) the secant method, 2) the modified secant method, 3) the spline fitting method, and 4) the total polynomial method. In the first method, the crack growth rate was computed by calculating the slope of a straight line connecting two adjacent a vs t data points and assigning the average crack growth rate at a point midway between each pair of data points. The second method was the modified secant method, which calculated the crack growth rate of two adjacent growth rates, also obtained by the secant method, and assigned the average crack growth rate to the middle point of the three data points. The third method was the spline fitting method, which fitted a smooth continuous curve through a set of data points by a third-order polynomial together with the requirement of a continuous first and second derivative at the data points. The crack growth rate was obtained by taking the derivative of the fitted function at the data point. The fourth method was the total polynomial method, which fitted an nth-order polynomial function to a set of data. Similar to the spline method, the crack growth rate was calculated by taking the derivative of the fitted function at the data point. In addition to using four different methods to calculate the crack growth rate, error analyses were conducted to determine the amount of error introduced into the crack growth data for each of the four crack growth rate calculation methods. In the error analysis, the crack length as a function of time was calculated by integrating $\dot{a}$ back and then comparing it with the original a vs t data. The relative error, expressed as the ratio of the difference between the integrated data and the original data to the original data, was calculated step by step, and the average af the relative errors was also calculated.[8] By comparing the average incremental errors, the effect of the $\dot{a}$ calculation method and the time interval used to measure the crack length vs time data on the accuracy of the calculated crack growth rate can be determined.

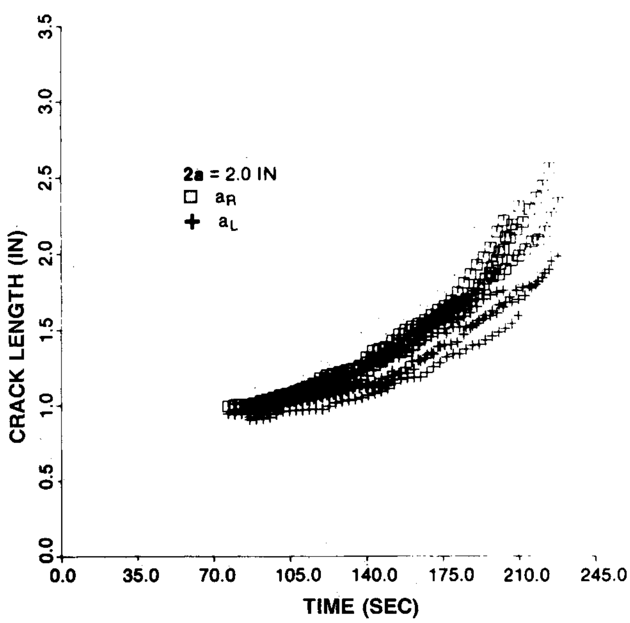

Fig. 2 Crack length vs time.

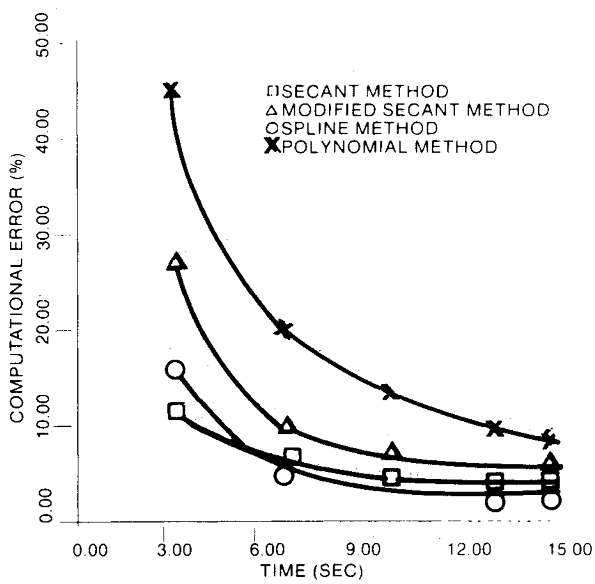

Fig. 3 Computational errors vs time.

A typical plot of crack length vs time data is shown in Fig. 2. These data were used to calculate the crack growth rates by the four different methods mentioned in the above paragraph. In addition, error analyses were conducted and the results of the analyses are shown in Fig. 3 with the computational error plotted as a function of the method of $\dot{a}$ calculation and the time interval Δt. The figure reveals that a definite trend exists between the computational error and the time interval Δt. For the four methods of $\dot{a}$ calculation considered, the computational error decreases with increasing Δt until it reaches a minimum value, and then it has a tendency to increase as the Δt is further increased. The figure also indicates that method 4 introduces the largest amount of error into the crack growth rate

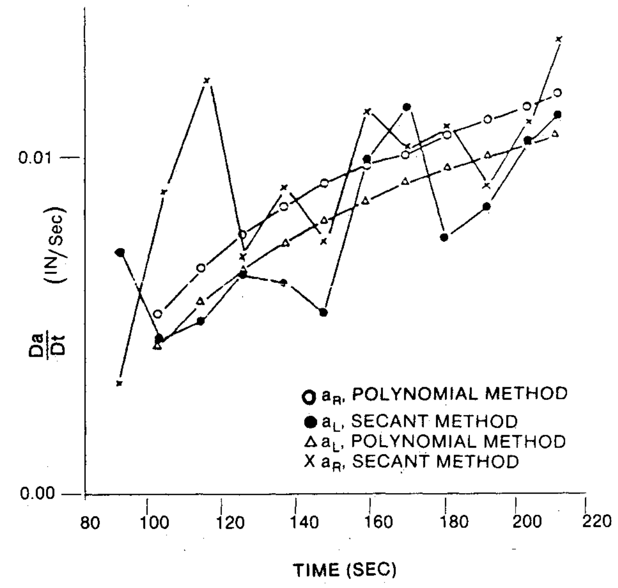

Fig. 4 Crack growth rate vs time.

followed by method 2, method 3, and method 1. The difference between the values of the computational error associated with method 1 and method 3 are small when Δt is greater than 6 s. However, the values of the computational error for method 4 are significantly higher than that of the other three methods. This indicates that the inherent variability of the crack growth rate may be masked by the data smoothing action introduced by fitting a smooth polynomial curve through the discrete data points. Therefore, when selecting a method to calculate the crack growth rate from the raw experimental data, the accuracy and the scatter that are introduced into the calculated crack growth rate by the selected data processing method should be considered. In addition, special attention should also be paid when selecting the time interval to record and analyze the raw experimental data in order to minimize the error and scatter in the calculated crack growth data.

To illustrate the effect of crack growth rate calculation method on the scatter of the calculated $\dot{a}$, two methods, secant and polynomial, which represent the two extreme cases, were used to process the raw data at 12-s time intervals. The results of the analyses are shown in Fig. 4. This figure clearly reveals that the method of $\dot{a}$ calculation has a significant effect on the scatter of the calculated crack growth rate. As illustrated in Fig. 4, the secant method produces a pronounced fluctuation of the crack growth rate. Although the fluctuation of $\dot{a}$ occurs during the entire time range of crack growth, the general trend for the crack growth is that, on the average, the crack growth velocity increases with increasing time. Referring back to Fig. 4, we recognize that the fluctuation of the crack growth rate, calculated by the secant method, does not occur when the polynominal method is used to calculate the crack growth rate. The smoothing action introduced by the polynominal method results in a continuous smooth crack growth velocity curve. It is interesting to point out that the crack growth rate calculated by the secant method oscillates approximately around the smooth crack growth rate curve obtained by the polynomial method. This phenomenon is consistent with what we expect because, in the polynomial method, a higher order polynomial function was selected to fit the discrete experimental data (crack length and time), resulting in a smooth curve. Therefore, it is reasonable to expect that, in general, the crack growth velocities calculated by the polynomial method may represent the mean crack growth rates with the crack growth rates calculated by the secant method oscillating around it.

Based on the above discussion, there is one important aspect revealed. It is found that the fluctuation of the crack growth rate is significantly affected by the method of $\dot{a}$ calculation. Based on experimental evidence, in general, the crack does not grow in a continuous and smooth manner. During the process of crack growth, both acceleration and deceleration as well as jump in crack growth were observed. Based on this experimental evidence and the results of the error analyses, it is believed that the secant method provides the best estimate of the actual crack growth history as well as the crack growth velocity.

When the crack growth rate fluctuates, there will be no one-to-one correspondence between the crack growth velocity and the stress intensity factor. This contradicts the current crack growth theory that requires a unique relationship between the stress intensity factor and the crack growth rate. The lack of uniqueness between the stress intensity factor and the crack growth rate raises the following questions: to what extent should the oscillation of $\dot{a}$ be taken into account and how should the data be presented and used. The answers to these questions depend on how the data will be used. If one is interested in knowing the central tendency of the crack growth behavior, the fluctuation in $\dot{a}$ seems unimportant and the polynomial method can be used to calculate the crack growth rate. However, if one is interested in determining the upper-bound limit on $\dot{a}$, the fluctuation in $\dot{a}$ should be considered in the data analysis, and the secant method is the proper method that should be used to calculate the crack growth rate.

Statistical Analysis

The presence of a crack in a stressed body leads to a redistribution of stresses near the crack tip. The stress field near the crack tip can be characterized by a single parameter known as the stress intensity factor K_I. The stress intensity factor has been widely used to characterize the crack growth behavior in various materials. The magnitude of the stress intensity factor at the tip of a crack in a solid propellant specimen subjected to a constant displacement rate is a function of material properties and the crack length. Since material properties and crack length are measured quantities, they are subject to measurement error and random error. Therefore, any error and uncertainty in the measurements will translate into error and uncertainty in the stress intensity factor. Hence, besides the scatter and error associated with the calculated crack growth rate, the calculated stress intensity factor is also subject to error and scatter within a specimen as well as among specimens. Under this condition, a precise statement on crack growth behavior in solid propellants cannot be made. Therefore, statistical methods must be used to treat the test data so the statistical variability of the measured data can be evaluated and statistical inference of the crack growth behavior can be made.

Typical plots of the crack growth rate $\dot{a}$ vs the mode 1 stress intensity factor K_I, for the four different methods of $\dot{a}$ calculation are shown in Figs. 5. According to these figures, the effect of the method of $\dot{a}$ calculation on the scatter of the crack growth data is clearly indicated. From these figures, we noted that Figs. 5a and 5b start at a higher value of K_I compared to the data shown in Figs. 5c and 5d. This is because the data shown in Figs. 5c and 5d are calculated at the time when the crack starts to grow, whereas the data shown in Figs. 5a and 5b are calculated at later times according to the method of $\dot{a}$ calculation mentioned before. In addition, for a given method of $\dot{a}$ calculation, these figures also indicate the scatter of the crack growth data due to the variability of the material property. The crack growth data shown in Figs. 5a and 5d were used in developing the regression model relating the crack growth rate and the stress intensity factor. It should be pointed out that the crack growth data in the initial crack growth stage were not used in the statistical analysis, and only the crack growth data in the stable crack growth stage were used in developing the regression model. A detailed discussion of the regression model is presented in the following paragraphs.

In Ref. 7, linear regression analyses were conducted to determine the equations relating the independent variable log K_I

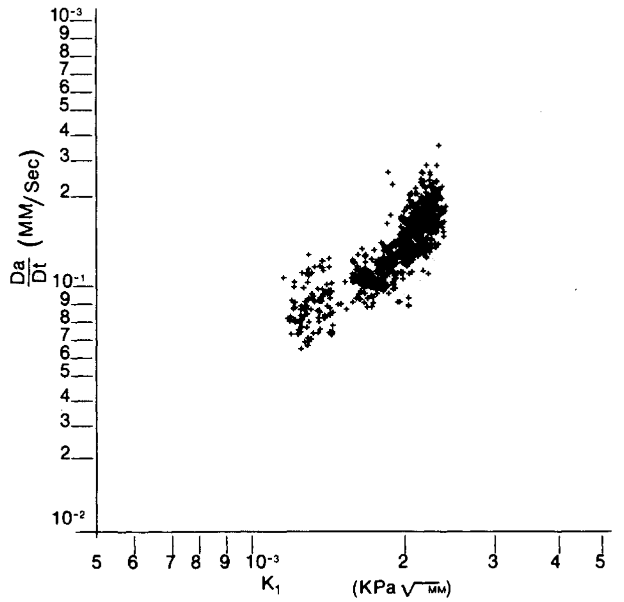

Fig. 5a Crack growth rate vs stress intensity factor, secant method.

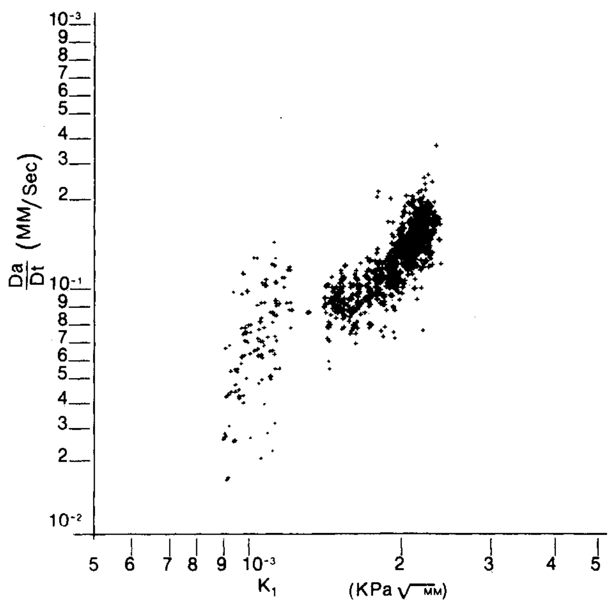

Fig. 5c Spline fitting method.

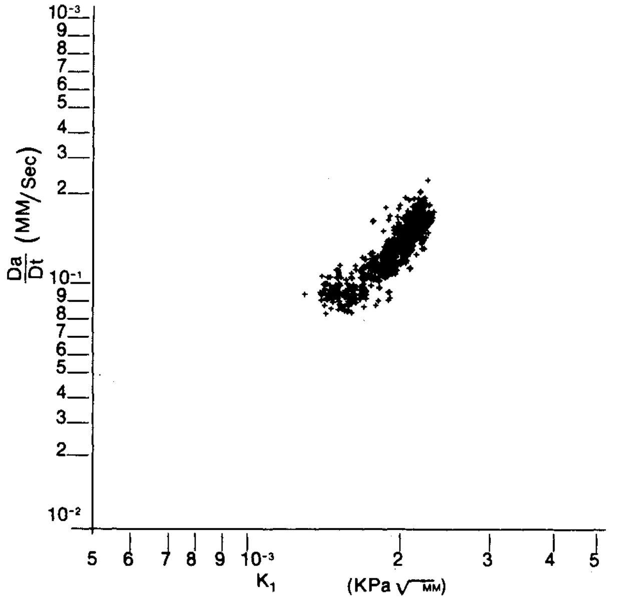

Fig. 5b Modified secant method.

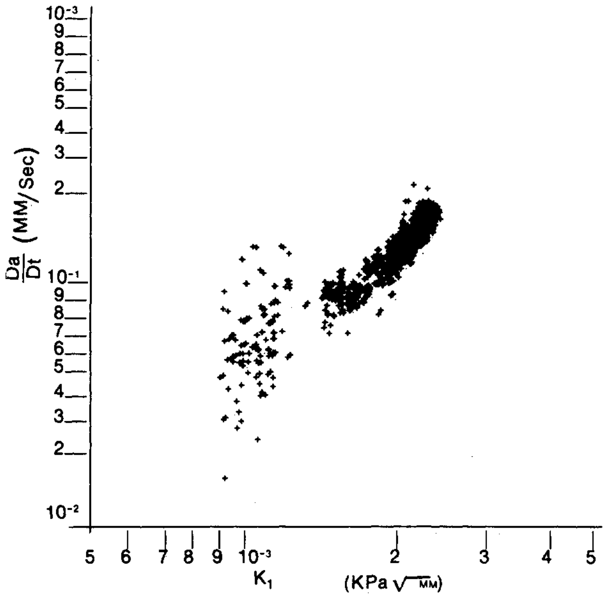

Fig. 5d Polynomial method.

to the dependent variable log $\dot{a}$. It should be mentioned that, for a given crack length, the variability of material properties among specimens induces scatter in the applied load, which in turn induces scatter in the calculated stress intensity factor. Therefore, in the regression analysis, depending upon how the stress intensity factor was determined, three different regression models were developed. For the first model, the regression equation was obtained by regressing log $\dot{a}$ on log K_I, ond this model was denoted as the R-model. For the second model, the regressing equation was obtained by regressing log $\dot{a}$ on log $\bar{K}_I$ where $\bar{K}_I$ was the average value of K_I for a given crack length, and this model was denoted as the M-model. For the last model, the regression equation was obtained by regressing log $\dot{a}$ on log $\hat{K}_I$ where $\hat{K}_I$ was the predicted value of K_I for a given crack length, and this model was denoted as the H-model.

In developing the three regression models, a linear regression analysis program was used to estimate the parameters associated with the three models. In the regression analysis, it was assumed that the statistical distribution function of the residuals of the transformed crack growth rate, log $\dot{a}$ is a normal distribution. To show the validity of this assumption, the residuals that were calculated from the three regression models were analyzed statistically. In the statistical analysis, Virkler's[9] statistical computer program was modified and used to determine the statistical distribution function of the residuals. The statistical distribution function considered are 1) normal, 2) two-parameter log-normal, 3) three-parameter log-normal, 4) three-parameter Weibull, and 5) three-parameter gamma. The statistical parameters were estimated by the use of the maximum likelihood estimators method and the graphical method. The goodness of fit of test data to a certain distribution function was determined by the use of a regression analysis, the chi-square test, and the Kolmogorov-Smirnov test. The result of the statistical analysis indicated that the normal distribution function fit reasonably well to the residual data as shown in Fig. 6. However, the estimated two parameters, slope β and intercept α, associated with the regression model as shown below might be biased.

$$\log \frac{da}{df} = \alpha_i + \beta_i \log K_I \qquad \text{where } i = \text{R,M,H} \tag{1}$$

This is due to the fact that in the linear regression model, the independent variable $\log K_I$ is subjected to both measurement error and random error. Under this condition, the standard assumption in the classical regression analysis, which requires that values of the independent variable be fixed, is violated. However, under certain restricted conditions on the relationship between the independent variable and the errors, as well as the variance of the errors, the classical regression model is still valid regardless of whether or not there are errors in the measured value of the independent variable.[10] Since the actual biases in α and β are indeterminable for the data considered, the Jackknife method,[11] which offers an asymptotically unbiased estimator, was used to estimate the values of α and β. The results of the analyses, shown in Table 1, reveal that the values of α and β that were estimated by the Jackknife method are close to those that were estimated by the regression method. The results also reveal that the values of α and β for the two different methods (secant and polynomial) of $\dot{a}$ calculation differ from each other. Since α and β and are empirically determined constants, it is expected that their values will vary from test to test as well as from one method of $\dot{a}$ calculation to another.

A plot of α vs β is shown in Fig. 7. As Fig. 7 clearly indicates, α and β are mutually dependent and related through the equation

$$\alpha = A - B\beta \tag{2}$$

and the computed correlation coefficients between α and β is 0.999. It is interesting to point out that the above equation is independent of the method af calculation. A similar phenomenon had been observed and reported in Ref. 5. The existence of Eq. (1) implies that all straight lines relating to $\log \dot{a}$ to $\log K_I$, obtained from the regression analysis, converge and meet at a point. In other words, a pivotal point exists around which these lines rotated. The coordinates of the pivotal point can be determined from the two constants A and B in Eq. (2). Since α and β are related through Eq. (2), it is expected that the decrease in α will result in an increase in β. Therefore, when using Eq. (1) to predict $\dot{a}$ for a given K_I, a different value of α does not necessarily result in a large difference in value of $\dot{a}$. The application of the developed regression model to predict crack growth rate for a given loading condition is discussed in the following paragraph.

Of the three regression models developed, the R-model would be a reasonable choice to use if there are no measurement errors in the values of independent variable K_I. If there are measurement errors in the values of K_I, M-model or H-model could be employed. These two methods will reduce the bias factor caused by the measurement error by smoothing the values of K_I. In the M-model, the smoothing is done by the simple arithmetic means, whereas in the H-model, the smoothed values of K_I are the predicted values of K_I that are obtained by regressing $\log K_I$ on $\log a$. For the data considered in Ref. 7, the H-model would be preferred to the M-model because the H-model directly accounts for the influence of the crack length on the stress intensity factor. However, as dis-

Table 1 Least squares and jackknife estimates of regression parameters

	$\alpha(S)^a$	$\beta(S)^a$	R^{2b}
Secant method			
R^c	−4.992 (0.140)	1.547 (0.080)	0.733
M^c	−5.183 (0.145)	1.657 (0.083)	0.747
H^c	−5.270 (0.145)	1.707 (0.083)	0.757
R^d	−4.986 (0.115)	1.544 (0.065)	
Polynomial method			
R^c	−5.403 (0.118)	1.758 (0.068)	0.832
M^c	−5.675 (0.106)	1.914 (0.061)	0.880
H^c	−5.791 (0.099)	1.980 (0.056)	0.900
R^d	−5.395 (0.132)	1.753 (0.074)	

[a]Standard deviation; [b]Correlation coefficient; [c]Least squares method; [d]Jackknife method.

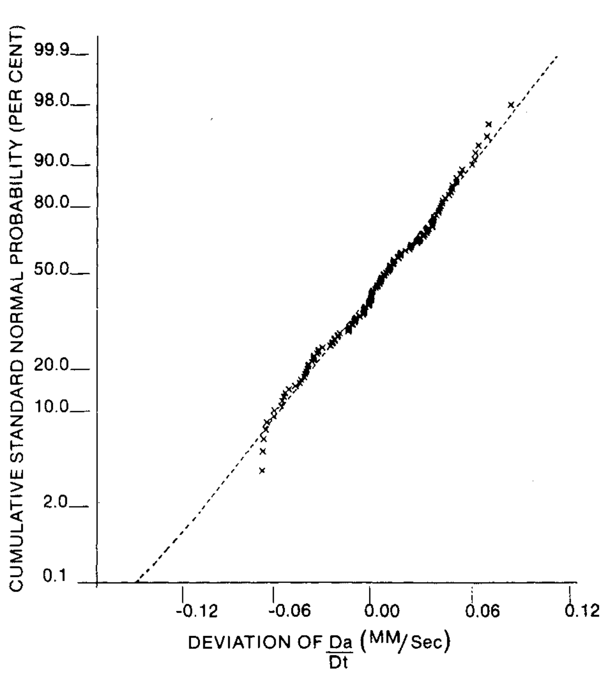

Fig. 6 Fit of residual data to the normal distribution.

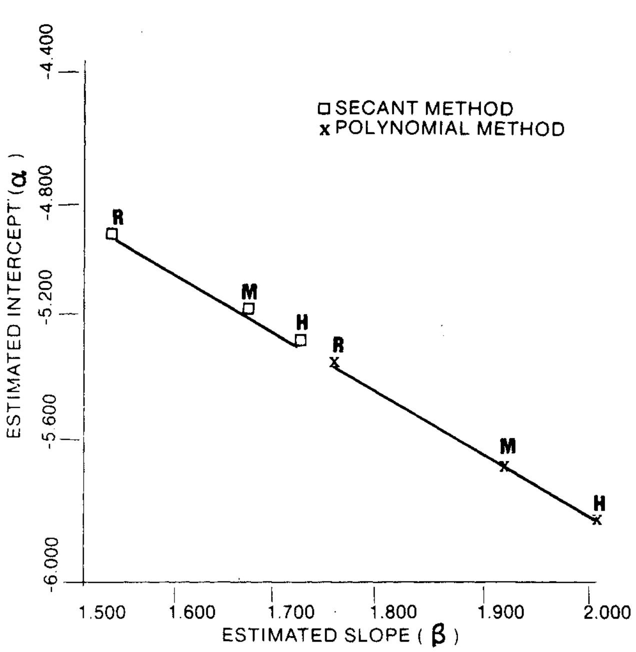

Fig. 7 Plot of α vs β.

cussed earlier, the small difference among the values of the α set and the β set in the three regression models may not produce significant differences in the predicted crack growth rate, and the validity of this statement is subjected to further investigation.

Conclusions

Based on the findings of this study, the following conclusions are drawn.

1) There is an optimum time interval that should be used to analyze the crack growth data in order to minimize the error in calculation.

2) In addition to the material variability, the method of $\dot{a}$ calculation can have a significant effect on the variability of the $\dot{a}$ data.

3) Data smoothing action introduces a large error but less scatter into the $\dot{a}$ data.

4) On one hand, if one is interested in knowing the central tendency of the crack growth behavior, the polynomial method should be used to calculate the crack growth rate. On the other hand, if one is interested in determining the upper-bound limit on the crack growth rate, the secant method should be used to calculate the crack growth rate.

5) The significance of the small difference among the estimated values of the α set and the β set in the regression models relating $\dot{a}$ to K_I needs to be investigated.

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